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Delay and fractional order parameter optimization based on optimization particle swarm optimization

Baozhong Ma, Chuanbo Ren, Jilei Zhou

***Abstract*— Time-delay vibration Control Technology is applied to the precision instrument workbench, whose influence is analyzed jointly by the control of fractional derivative and delayed feedback. Delay feedback control with fractional derivative is considered the workbench modeling. The stability switch theory is used to analyze the stability of the delayed dynamical systems, and find the system stable region. In the frequency domain, the objective optimization function is established based on the frequency domain. The delay feedback control parameters are optimized by the PSO (particle swarm optimization) and we can get the optimized control parameters. What's more, the system vibration response simulation analysis is taken under time domain. Simulation results show that the fractional derivative and delay vibration control technology combined with the active suspension system can achieve good damping effect under harmonic excitation and random road excitation. Simulation results show that fractional derivative and delay vibration control technology combined with the workbench is less than the without fractional derivative and delay vibration control technology combined value is reduced by 99%, The value of the velocity and acceleration feedback is reduced by 99% and 68% under harmonic excitation. It shows that the fractional derivative and delay vibration control technology combined with the precision instrument workbench can effectively reduce vibration and provide a theoretical basis for active control technology.**

***Index Terms*— Fractional; Workbench System; Time-delay; Stability; Particle Swarm Optimization (PSO)**

I. INTRODUCTION

In the aerospace, automotive industry, instrumentation, weapons, construction and engineering machinery and other fields, precision instrument workbench has a wide range of use. As the external environment and equipment work always produces vibration, vibration on the performance of the instrument causes bad impacts, so the normal use of the equipment and vibration-damping technology which is closely related to effective ways to reduce vibration is particularly important.

In recent years, the theory and application of time-delay control and fractional-order calculus have become a hot topic at home and abroad. They have attracted the attention of many research institutions and scholars, and have obtained a large amount of research results. In 1992, Olgac, McFarland and Holm-Hansen [1] first proposed the concept of time delay dynamic vibration absorber. Subsequently, Olgac [2] has designed a dual-frequency time delay dynamic vibration absorber which can be given a completely fixed dual-frequency inhibiting the vibration of the main system. Jalili and Olgac [3] solved the stability of the time-delay vibration system by using the method of graphic setting. Olgac , Renzulli [4] and many people realize the time delay dynamic vibration absorber can achieve parameter real-time control. Xu and Zhao [5-6] studied the time delay nonlinear dynamic vibration absorber and the nonlinear dynamic vibration absorber. The result shows that the stability control range of the nonlinear time delays, dynamic vibration absorber is better than that of the linear time delay dynamic vibration absorber wider range. Zhang Wenfeng, Hu Haiyan [7] and others studied the time delay factors in the vehicle suspension 'ceiling' damping control. Compared with the integer order, fractional calculus is the main advantage in describing the system memory and heredity. Deng and Li [8] studied the robust stability of fractional systems with multiple delays. Mihailo and Lazarevic et al. [9-10] used Bellman-Gronwall theorem to solve the stability problem of time-delay fractional-order system in a finite time interval. Chen and Moore [11-12] used the Lambert W function to analyze the stability of fractional order systems with time delay. Based on the principle of the argument and the Hassard principle of the time-delay system, the stability criteria of the fractional-order dynamic system with time delay are proposed by Wang et al. [13] This paper provides a theoretical basis for the application of fractional derivative and active damping control technology to the damping of the precision instrument table.

In this paper, we study the influence of fractional derivative and time delay damping control on the vibration of a precision instrument table with active damping control technology. The purpose of this paper is to improve the performance of the workbench more comprehensively by increasing the number of control parameters, adding time-delay state feedback with fractional derivative in the workbench system, establishing frequency-domain characteristics of the ground excitation according to the table acceleration. The objective function of the system vibration is obtained. The control parameters are selected through optimization analysis. The optimal feedback coefficient, the time delay and the fractional order are obtained, and the simulation is carried out in the time domain.

II. MECHANICAL MODEL

Considering the structure of the workbench system and the form of vibration, neglecting the degrees of freedom in the pitch and roll directions of the system, only consider the vertical vibration workbench. The system is simplified as a two-degree-of-freedom system consisting of a workbench and a vibration absorber.

Tab. 1 workbench model parameters

quality of the platform	quality of the vibration absorber	stiffness co efficient of the platform	stiffness coefficient of the vibration absorber	damping coefficient of the platform	damping coefficient of the vibration absorber
m_1/kg	m_2/kg	$k_1/N.\ m^{-1}$	$k_2/N.\ m^{-1}$	$c_1/N.\ s.\ m^{-1}$	$c_2/N.\ s.\ m^{-1}$
150	15	15000	1500	30	3

According to the purpose of the study, simplify the workbench, and establish the mechanical model as shown below:

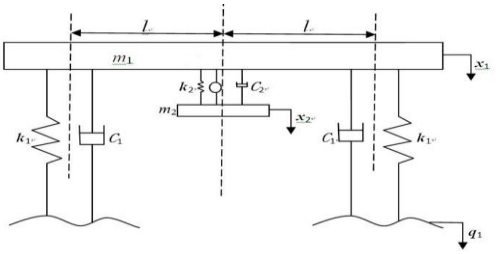


Fig.1. 1/2 workbench model with time delayed feedback control based on fractional derivative

Let x_1, x_2 be the generalized coordinates, and obtained the differential equations of the system.

$$\begin{cases} m_1\ddot{x}_1+2c_1(\dot{x}_1-\dot{q})+2k_1(x_1-q)-k_2(x_2-x_1)-c_2D^{\alpha}(x_2-x_1)-gx_1(t-\tau)=0 \\ m_2\ddot{x}_2+k_2(x_2-x_1)+c_2D^{\alpha}(x_2-x_1)+gx_1(t-\tau)=0 \end{cases} \quad (1)$$

m1 is the quality of the workbench, and m2 is the quality of the vibration absorber. k_1, c_1, respectively, is the stiffness coefficient and damping coefficient of the workbench. k_2, c_2, respectively, is the stiffness coefficient and damping coefficient of the vibration absorber. g is the gain coefficient of displacement feedback. τ is delay. D^{α} is the fractional differential operator. The workbench specific parameter values as showed in figure 1. The design of dynamic vibration absorber parameters is based on the basic principle of the dynamic vibration absorber. When the dynamic vibration absorber resonates with the external excitation frequency acting on the main system, the dynamic absorber absorbs energy to make the main system have the best vibration-damping effect. Assume that the systems are motivated by the periodic excitation, q1= sin (14t). x_1, x_2, respectively, is the vertical displacement of the workbench and the vibration absorber.

III. SYSTEM STABILITY ANALYSIS

A. System stability conditions

The Laplace transform is applied to the differential system equations:

$$\begin{bmatrix} m_1s^2+2c_1s+2k_1+k_2+c_2s^{\alpha}-ge^{-\tau s} & -k_2-c_2s^{\alpha} \\ -k_2-c_2s^{\alpha}+ge^{-\tau s} & m_2s^2+k_2+c_2s^{\alpha} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 2c_1sQ+2k_1Q \\ 0 \end{bmatrix} \quad (2)$$

Where, X_1, X_2, and Q, respectively, is the Laplace transforms of x_1, x_2, and q.

To get the characteristic equation of the system:

$$CE(s)=\begin{vmatrix} m_1s^2+2c_1s+2k_1+k_2+c_2s^{\alpha}-ge^{-\tau s} & -k_2-c_2s^{\alpha} \\ -k_2-c_2s^{\alpha}+ge^{-\tau s} & m_2s^2+k_2+c_2s^{\alpha} \end{vmatrix}=0 \quad (3)$$

Including, α∈ (0, 1), because of continuing $e^{-\tau s}$ beyond the exponential function, analyzed the stability of the system using stability switch method.

Equation (3) is transformed into as follows:

$$CE(s^{1/n})=P(s^{1/n})+Q(s^{1/n})e^{-\tau s}=0 \quad (4)$$

n is the common denominator of the exponential power of complex variables. The function of P, Q as follows:

$$P(s^{1/n})=m_1m_2s^4+2m_1c_1s^3+(2m_2k_1+m_2k_2+m_1k_2)s^2+2c_1k_2s$$
$$+2k_1k_2+(c_2m_2+m_1c_2)s^{2+\alpha}+2c_1c_2s^{\alpha+1}+2k_1c_2s^{\alpha}$$
$$Q(s^{1/n})=-m_2gs^2+gk_2$$

According to the system stability and sufficient conditions when all the characteristic roots s_i (i = 1,2, ..., n) of the system have negative real parts (in the left half plane of the [s] plane). That is $|arg(s_i)|>\pi/2$. The system is stable, as showed in figure 2.

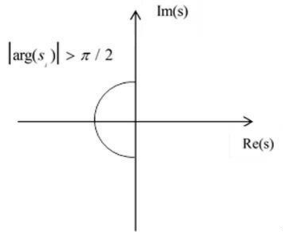


Fig. 2 the stability region of integral order system

Introduce an auxiliary complex variable $\lambda=s_1/n$, and the characteristic equation with the fractional power in s domain is transformed into the characteristic equation of integrative power in λ domain. The characteristic equation is written as follows:

$$CE(\lambda)=P(\lambda)+Q(\lambda)e^{-\tau\lambda^n}=0 \quad (5)$$

According to the characteristic equation in λ complex plane, the system can obtain the stable region, $|arg(s_i)|>\pi/2$. As showed in figure 3.

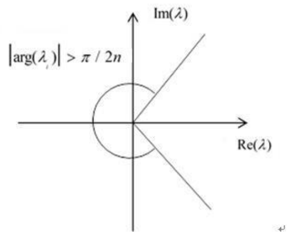


Fig. 3 the stability region of fractional order system

It can be seen that the stability region of the fractional-order system is larger than the stability region of the integer-order system. Therefore, the system control parameters have more intervals for adjustment. The system has better robustness.

B. System Stability Criteria

Firstly, the Rekasius substitution is used to transform the characteristic equation containing the transcendent term into an ordinary algebraic equation, the Rekasius as follows:

$$e^{-\tau s}=\frac{1-Ts}{1+Ts}=\frac{1-T\lambda^n}{1+T\lambda^n} \quad (6)$$

$$\tau=\frac{2}{\omega}(\tan^{-1}(\omega T)+k\pi) \quad k\in\square, T\in R \quad (7)$$

By equation (5) and (6), we can obtain the characteristic equation as follows:

$$Ce(\lambda)=P(\lambda)(1+T\lambda^n)+Q(\lambda)(1-T\lambda^n) \quad (8)$$

Let $s=j\omega$, then $\lambda=\omega^{1/n}e^{j\pi/2\pi}$. Substitute $\lambda=\omega^{1/ne}j^{\pi/2\pi}$ into the

equation (8), and separated the real part and imaginary part. And get the equation.

$$Ce(\omega^{1/n}e^{j\pi/2n}) = Ce_{\Re}(\omega^{1/n}e^{j\pi/2n}) + jCe_{\Im}(\omega^{1/n}e^{j\pi/2n}) \quad (9)$$

Let the real and imaginary parts equal zero. We can get the simultaneous equations of T and ω.

$$\begin{cases} Ce_{\Re}(\omega^{1/n}e^{j\pi/2n}) = 0 \\ Ce_{\Im}(\omega^{1/n}e^{j\pi/2n}) = 0 \end{cases} \quad (10)$$

If there is not a real solution of equations (10), the delay of the motion trajectory of change will not make the system characteristic root to the imaginary axis; if there is a real solution, existing system characteristic root is located in the imaginary axis.

By equations (7) all the critical time delay system can be obtained.

The traversal direction of the root trajectory of the system at the critical characteristic root is judged by the characteristic root movement trend $RT|_{s=j\omega i}$, the characteristic root movement trend is denoted as follows:

$$RT|_{s=j\omega_i} = \operatorname{sgn}\left(\Re\left(\frac{ds}{d\tau}\bigg|_{\substack{s=j\omega_i \\ \tau=\tau_i}}\right)\right) \quad (1)$$

Where,

$$\frac{ds}{d\tau}\bigg|_{s=j\omega_i} = \frac{sQ(s)\mathrm{e}^{-\tau s}}{P'(s)+(Q'(s)-\tau Q(s))\mathrm{e}^{-\tau s}}\bigg|_{\substack{s=j\omega_i \\ \tau=\tau_i}}$$

If $RT|_{s=j\omega_i}=-1$, the number of the system is reduced by two unstable roots. If $RT|_{s=j\omega_i}=+1$, the number of the system is increased by two unstable roots.

When the delay is zero, using the equation $|\arg(si)|>\pi/2$ to judge whether the system is stable. At the time delay is not equal to zero. System stability is closely related to the solution of equations (10). If equation (10) does not have a real solution, the system is stable with full time delay. If the system (10) has a real root, the system will have a critical characteristic $s_i=j\omega_i$ on the imaginary axis, and there exists a critical time lag accordingly $\tau_{ik}= (2/\omega_i)\,(tan^{-1}(\omega_i T) + k\pi)$, and then at the critical delay, according to the trend of the characteristic root movement $RT|_{s=j\omega i}$ judge whether the system trajectory through the imaginary axis, so as to judge the critical time delay interval of the system stability.

IV. Control Parameter Optimization

Particle Swarm Optimization (PSO) [14-16] is an adaptive stochastic optimization algorithm based on search strategy. As the optimization algorithm is simple and effective, the convergence speed is quick, and the parameter setting is few. There is no centralized control constraint, and it has strong robustness of the system. So it is widely used in multi-objective function optimization and intelligent system control. The particle swarm optimization (PSO) algorithm is combined with the time-delay fractional order damping, and the amplitude-frequency characteristic function of the workbench system is used as the objective function, and solves the optimal value.

According to the needs of the actual algorithm, the three-dimensional search space is established to represent the gain, time delay and fractional order respectively. The particle swarm is generated separately, considering the large gap between the parameters to be optimized X= (X1, X2,⋯ Xn), Y=(Y_1,Y_2,…Y_n), Z =(Z_1,Z_2,⋯Z_n). The particles of the search space are denoted as vectors, respectively, $X_i= (x_{i1}, x_{i2})^T$, $Y_i= (y_{i1}, y_{i2})^T$, $Z_i= (z_{i1}, z_{i2})^T$, and calculate the fitness value corresponding to each particle position. The velocity of the i-th particle is $V_i = (V_{i1}, V_{i2})^T$. During the each iteration, the particle updates its speed and position through the extremity of the individual and the global extreme, updating the formula:

$$\begin{cases} V_{id}^{k} = \omega V_{id}^{k-1} + c_1 r_1\left(pbest_{id} - x_{id}^{k-1}\right) + c_2 r_2\left(gbest_{id} - x_{id}^{k-1}\right) \\ X_{id}^{k} = X_{id}^{k-1} + V_{id}^{k-1} \end{cases}$$

Where, ω is inertia weight. c1 and c2 are acceleration constants. Adjust the maximum learning step. r1 and r2 are random functions to increase the randomness of a random search. In the process of optimization, in order to avoid the blindness of searching the optimal particles, according to the optimum conditions and actual conditions of the objective function, set the reasonable range of gain, delay and fractional order. First use *maple* to calculate |H (ω) |, and then bring into Matlab using particle swarm optimization algorithm and get feedback gain g and delay τ and α.

In order to facilitate the numerical analysis, and introduced the dimensionless parameter.

$$t^* = \sqrt{\frac{k_1}{m_1 t}}, \tau^* = \sqrt{\frac{k_1}{m_1 \tau}}, \alpha_1 = \frac{m_1}{m_2}, \beta = \frac{c_1}{c_2}, \gamma = \frac{k_1}{k_2}, c = \frac{c_1}{\sqrt{k_1 m_1}}, g_k = \frac{g}{k_1}, \omega_1 = \sqrt{\frac{k_1}{m_1}}$$

The differential equation is written as a Laplace transform.

$$\begin{cases} X_1 s^2 + 2cs(X_1 - \mathrm{Q}) + 2(X_1 - \mathrm{Q}) - \gamma(X_2 - X_1) + \beta c s^{\alpha}\frac{1}{\omega}(X_2 - X_1) - g_k X_1 e^{-\tau s} = 0 \\ X_2 s^2 + \beta\alpha_1 c\frac{1}{\omega}(X_2 - X_1)s^{\alpha} - \gamma\alpha_1(X_2 - X_1) + g_k X_1 e^{-\tau s} = 0 \end{cases} \quad (12)$$

Where, s=jω.

The optimization objective function is Amplitude - Frequency characteristics of workbench for pavement excitation.

$$\min|\mathrm{H}(\omega)| = \left|\frac{X_1(\omega)}{Q(\omega)}\right| = \left|\frac{-A_{22}(2+2cs)}{A_{11}A_{22} - A_{12}A_{21}}\right| \quad (2)$$

$$s.t \begin{cases} g \geq 0 \\ 0 \leq \tau \leq 1 \\ 0 \leq \alpha \leq 1 \end{cases}$$

Where

$$A_{11} = s^2 + 2cs + 2 + \gamma - \beta c s^{\alpha} - g_k e^{-\tau s}, \quad A_{12} = \beta c\frac{1}{\omega}s^{\alpha} - \gamma$$

$$A_{21} = \gamma\alpha_1 + g_k e^{-\tau s} - \beta\alpha_1 c\frac{1}{\omega}s^{\alpha}, \quad A_{22} = \beta\alpha_1 c\frac{1}{\omega}s^{\alpha} - \gamma\alpha_1 + s^2$$

The result is shown as follows:

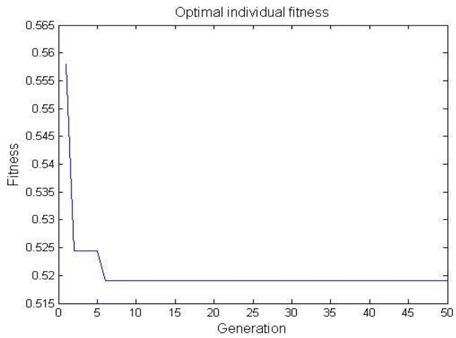


Fig. 4 Fitness function iterative optimization

By analyzing the impact on the objective function, we can get a set of optimization parameters α = 0.4, g = 9986.1 N/m, τ = 0.1s.

Providing that the time delay is zero, the characteristic equation of the fractional time-delay system degenerated to the characteristic equation of the general fractional-order system. To make the system stable, it should be met

$$|arg(\lambda_i)|>\pi/2n=\pi/4 \quad (14)$$

Where $n=\alpha^{-1}=2.5$.

Substituting α = 0.4, g = 9986.1 N/m, τ = 0.1s into the characteristic equation (5) and get λ_i:

$$\lambda_{1,2} = 6.3415 \pm i5.1125 \text{，} \lambda_{3,4} = 1.4182 \pm i2.4182$$

$$\lambda_{5,6} = \text{-}6.0152 \pm i6.1525 \text{，} \lambda_{7,8} = \text{-}2.2319 \pm i1.1255$$

It is proved that the characteristic root satisfies (14). The system is stable when the time delay is zero.

When the time delay is 0.1s, the optimized control parameter α = 0.4, g = 9986.1 N/m is substituted into the system of equations (10). Use the Maple software to calculate the solution (T, ω), where the value of ω as follows:

$$\omega_{1,2} = \text{-}5.3414 \pm i2.2518 \text{,} \quad \omega_{3,4} = \text{-}69.8681 \pm i15.6601$$

$$\omega_{5,6} = 7.4253 \pm i1.2060 \text{,} \quad \omega_{7,8} = 69.2512 \pm i5.5239$$

We can see the real domain without solution in equations (10), the system does not have the characteristic root of critical stability. Correspondingly, there is no stable critical time delay. This indicates that the change of time delay does not cause the trajectory to traverse the imaginary axis of the complex plane, and does not cause the system stability to change. Since the system is stable at zero time delay, the system is stable over the entire time-delay interval. This shows that the selected optimal control parameters are feasible.

V. VIBRATION RESPONSE ANALYSES

In order to test the effect of fractional order delay damping in the system vibration reduction, and by excluding the workbench of a class of linear fractional order vibration absorbing system for comparison. Analysis of frequency - domain characteristics of time - delayed fractional - order absorber system and time domain response of vibration damping effect.

A. Frequency Domain Characteristics

To optimize parameters of α = 0.4, g = 9986.1 N/m, τ = 0.1s into the amplitude, frequency characteristics of vibration response function, and get the figure 5.

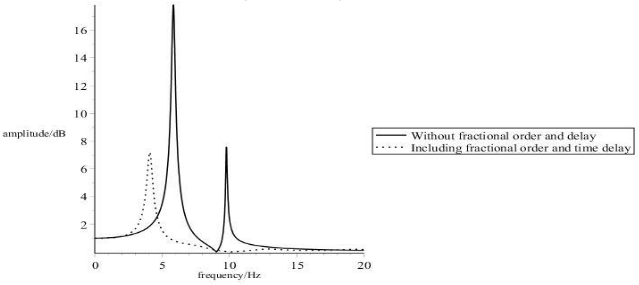
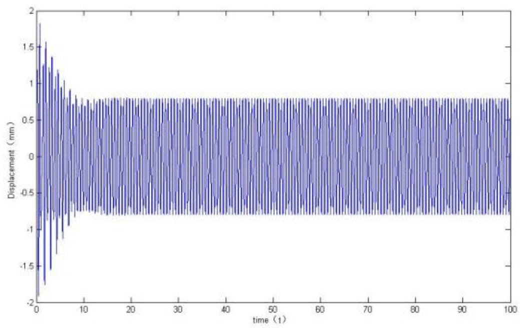


Fig. 5 feedback frequency domain

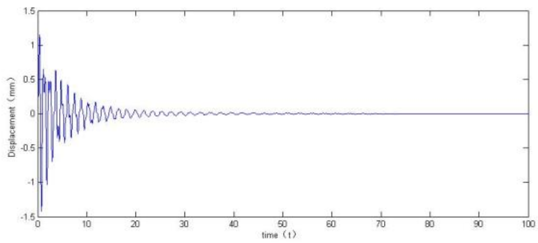
From the figure, in the frequency range of 4 ~ 15Hz, because of the time delay, it has a damping effect. It can change the vibration phase, thereby reducing the vibration amplitude of the main system. Fractional state feedback can not only adjust the damping force, but also adjust the elastic restoring force. At the same time, it can adjust the control force size and phase, so that the system to achieve the optimal damping effect. The general advantages of active vibration absorber with fractional time delay dynamic absorber. That can achieve real-time tuning. When the passive dynamic vibration absorber reduced vibration absorption effect due to the system frequency changes, the fractional time delay dynamic vibration absorber can be tuned, so that the main system to achieve the minimum vibration. From this can be drawn that compared with the passive system, the active system with fractional order time-delay feedback control can reduce the vibration of the workbench. If we reasonably control the delay and fractional size, it will match with the feedback system. The vibration of the system can be significantly reduced. It is shown that the time-delay fractional-order dynamic vibration absorber has a great advantage over the traditional passive vibration absorber in a frequency band.

B. Time Domain Simulation

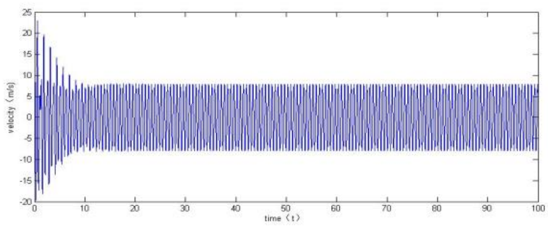
Passive feedback systems without fractional order and time delay are respectively compared with the active feedback system with fractional order and time delay. Analyze the time domain simulation of vibration response under the action of harmonic excitation (q=sin (14t)). The results are shown in Figure 6.



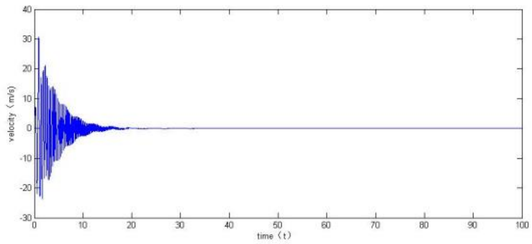
a) Workbench displacement response without time delay and fractional



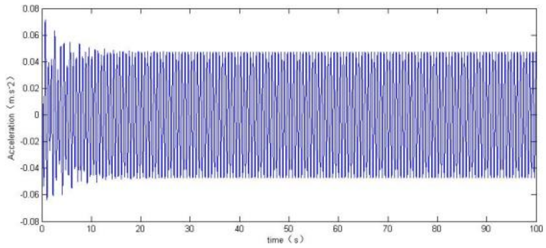
b) Workbench displacement response with time delay and fractional



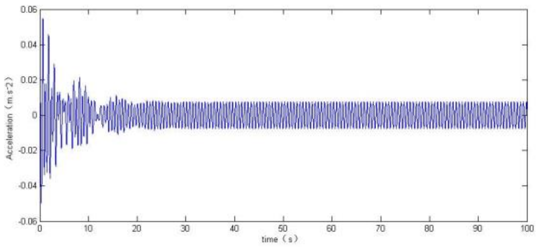
c) Workbench velocity response without time delay and fractional



d) Workbench velocity response with time delay delay and fractional



e) Workbench acceleration response without time delay and fractional



f) Workbench acceleration response with time delay and fractional

Fig.6 The simulation comparison chart of the workbench stability index

The simulation shows that in the frequency range of 4 ~ 15Hz, the time-delay fractional-order dynamic vibration absorber has a great advantage over the traditional passive vibration absorber. Including delay fractional order feedback displacement decrease 99% than without delay and fractional order feedback, the vibration of the main system is almost completely absorbed, the velocity feedback is reduced by 99% and the acceleration feedback is reduced by 68%, as showed in table 2. This indicates that when the system is stimulated by the external incentive, the fractional-order time delay dynamic vibration absorber has the ability to make the external excitation fast decay performance, and the attenuation effect is very obvious. Therefore, the fractional derivative and delay damping active control technology is applied to the damping of the precision instrument workbench, controlling the feedback gain, delay, differential operator value, can effectively reduce the vibration.

Table2. The workbench stability index

Vibration response values	displacement feedback (mm)	velocity feedback (m/s)	acceleration feedback (m/s^{-2})
Without delay and fractional feedback	0.7525	7.8579	0.0536
With delay and fractional feedback	0.0023	0.0022	0.0174

VI. Conclusion

In this paper, the theory of time-delay fractional-order dynamic vibration absorber is used to construct the model of precision instrument workbench. Fusing the theory of fractional - order dynamic vibration absorber and particle swarm optimization. The time - delay and fractional - order feedback control is optimized and the model is simulated numerically. The following conclusions are obtained:

Firstly, for the precision instrument workbench system, the active feedback control is introduced by using the time - delay and fractional - order dynamic vibration absorber theory. The stability model of the workbench is established and stability analysis is done by using the stability switching principle, and obtained the system stability interval, and the results show that the optimization results satisfy the system stability condition.

Secondly, based on particle swarm optimization (PSO) algorithm, the time - delay and fractional - order vibration parameters are optimized to optimize the objective function with the amplitude - frequency response of the workbench. The simulation shows that active systems with fractional-order time-delay feedback control can be tuned in real time, and adjusts the phase and control the size. Under the simple harmonic excitation, in the frequency range of 4 ~ 15Hz, including delay fractional order feedback displacement decrease 99% than without delay and fractional order feedback, the vibration of the main system is almost completely absorbed, the velocity feedback is reduced by 99% and the acceleration feedback is reduced by 68%. It is shown that the time-delay fractional-order dynamic vibration absorber has a great advantage over the traditional passive vibration absorber. It can significantly improve the vibration absorber damping performance, so that the main system can achieve the minimum vibration.

Third, it is a relatively new research method to apply the fractional calculus theory and time delay damping technology in the vibration reduction project of the precision instrument table, and it is also the main innovation of this paper. The results of this paper prove that the method is feasible and effective in theory, and has certain engineering value.

Acknowledgment

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Baozhong Ma, He was born on 28th September, 1990 in Shandong province, China. He is a graduate student at the Shandong University of Technology, and major in mechanics. His research direction is the engineering mechanics optimization and vehicle system dynamics.


Chuanbo Ren , He was born on 1964 in Shandong province, China. He is a professor and PhD Tutor at the Shandong University of Technology. His main research direction is he engineering mechanics optimization and vehicle system dynamics.


Jilei Zhou, He was born on 30th August, 1982 in Shandong province, China. He is a doctor at the Shandong University of Technology. His main research direction is he engineering mechanics optimization and vehicle system dynamics.